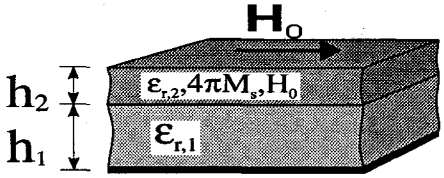
TABLE I

NORMALIZED PROPAGATION CONSTANTS OF THE PROPER AND VARIOUS IMPROPER MODES ($0 < \mathrm{Im}(k_z/k_0) < 11$, $0 < \mathrm{Re}(k_z/k_0) < 5$) FOR A GROUNDED WAVEGUIDE WITH A DIELECTRIC-FERRITE COMPOSITE SUBSTRATE. LOWER DIELECTRIC LAYER WITH h_1=2mm, h_2=1mm, AND BIASED FERRITE LAYER ($\mathbf{H}_0 = H_0\mathbf{a}_x$) WITH $\epsilon_{r,1} = 12.9$, $\epsilon_{r,2} = 12.6$, $4\pi M_s = 2750$ G, $H_0 = 8\ 25$ Oe, Freq = 20 GHz.



Proper Modes

	$\mathrm{Re}(k_z/k_0)$
# 1	3.3705
# 2	2.7107
# 3	1.1443

Improper Modes

	$\mathrm{Re}(k_z/k_0)$	$\mathrm{Im}(k_z/k_0)$
# 1	3.3527	0
# 2	0.0711	-10.6579
# 3	0.0748	-7.9868
# 4	0.0923	-5.1117
# 5	0.4336	-1.2679
# 6	0.8296	-9.2161
# 7	1.1063	-4.4223
# 8	1.1500	-1.0485
# 9	1.1794	-10.7319
# 10	1.4345	-6.8996

GHz. The dispersion curves of these modes for frequencies below 20 GHz are shown in Fig. 3. The solid lines represent its frequency behavior when the corresponding root lies on the proper sheet; the dashed and dotted lines represent its mathematical prolongation on the improper sheet. If the *cutoff frequency* is defined as that frequency where the root of one mode passes through the branch cut, Fig. 3 shows that the fundamental TM slab-guided wave (marked by #1) has no cutoff frequency, but the other two slab-guided modes show cutoff frequencies at 7.35 and 14.55 GHz, respectively. Note that in open structures, the cutoff frequency separates the nature of the mode into proper and improper, rather than into propagating and evanescent. If Fig. 3 is read from the right- to the left-hand side, we can observe how the proper real mode #3 becomes an improper real mode at its cutoff frequency. This improper real mode encounters another improper real mode coming from high values of $\mathrm{Re}(k_z/k_0)$ at 8.7 GHz, and these two modes come together to form a complex improper mode below this frequency. The transition between the physical bound and unbound modes (that is, the proper real and the leaky modes) is made throughout the nonphysical real improper mode. This type of conduct has been previously reported in the literature and is usually known as *spectral gap* [9]. The above behavior is also found for mode #2, but it appears beyond the limits of Fig. 3. Moreover, the scheme described for mode #3 is always found for all slab-guided modes of grounded layered waveguides.

IV. SUMMARY

This work presented an efficient numerical procedure to compute the propagation constants of both the proper and improper modes of a planar bianisotropic layered waveguide bounded by upper and bottom interfaces which can be simulated by impedance/admittance dyads. The dispersion relation of the waveguide has been posed, in a compact way, as the roots of a certain analytic (no poles or branch cuts) function, and integral techniques are suggested to search efficiently for these roots. The transition from proper to improper modes in a grounded waveguide containing a biased ferrite layer has been studied, and a spectral gap has been found in the prolongation from the bound mode to the leaky mode.

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A Note on the Mode Characteristics of a Ferrite Slab

Hung-Yu Yang

***Abstract*—Properties of guided-wave modes of a ferrite slab propagating in the direction transverse to the bias field are reexamined. Analytic results for the frequencies where magnetostatic and dynamic modes exist simultaneously are found. The method of eliminating the dynamic modes in the magnetostatic-wave operation is described. The formulas for the distinction of oscillatory and surface-wave modes are also derived.**

I. INTRODUCTION

Guided-wave properties of a ferrite slab have been studied extensively in the past, for example with a magnetostatic analysis [1]–[5] and with a full-wave analysis [6]–[10]. It has been found that microwave devices with ferrite slabs are capable of space-frequency selection of signals [11]. It has been well recognized that a ferrite

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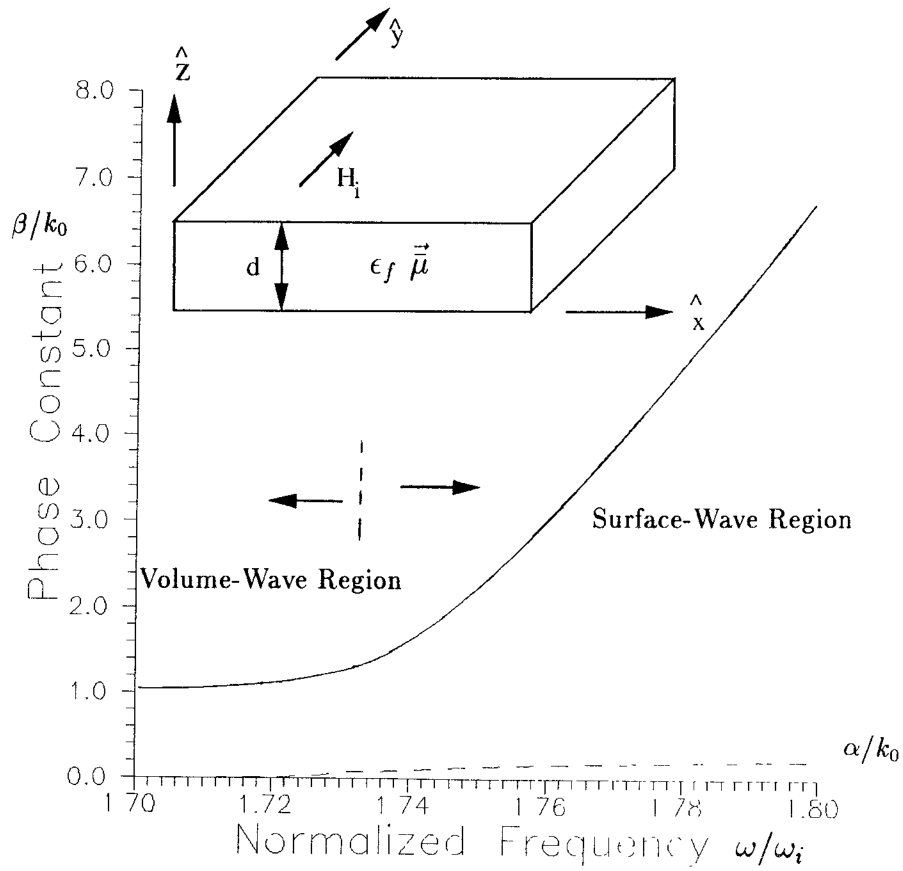

Fig 1. Dispersion relation for the fundamental (magnetostatic) mode. H_i = 500 Oe, $4\pi M_0$ = 1000 Gauss, ΔH = 10 Oe, d = 0.4 mm, ϵ_f = 14.

slab supports magnetostatic surface modes if the guided waves are in the direction transverse to the in-plane bias field. For such a case, the magnetostatic analysis predicts that within a frequency range, there exist a surface wave mode (wave decaying exponentially from the air-ferrite interface). Also, for lower frequencies, there exist infinite number of volume-wave modes (waves are oscillatory within the ferrite slab). The magnetostatic analysis also predicts a mode discontinuity from a volume wave to a surface wave. It has been found with a full-wave analysis that the TM modes are unaffected by the bias field and are identical to the modes of a dielectric slab. The modes corresponding to the magnetostatic waves are TE modes. With a full-wave analysis, it has been revealed that there may exist an additional TE mode [6] that is not found from a magnetostatic method. For magnetostatic wave applications, this dynamic mode is a higher-order mode and is often undesirable. Although the full-wave approach has been well developed, the clarification of the mode properties is not complete. Parekh [7] discussed the distinction of surface and volume wave modes for a grounded ferrite slab. It had been found that for grounded ferrite slabs, there is no mode cut-off and the magnetostatic modes and the dynamic modes do not exist simultaneously. On the other hand, for a ferrite slab, the fundamental (TE) mode has no cut-off and encounters infinite mode discontinuities near ferrite resonance. These mode discontinuities can be identified analytically, but they won't exist physically. This may be proven by introducing a small loss to the structure.

In this paper, the properties of guided-wave TE modes of a ferrite slab are reexamined. The aim is to provide an understanding of the properties of modes that have not been discussed previously. The analytic formula for the cut-off frequency for the dynamic TE modes and the method of eliminating this undesirable mode at the frequency of interest is discussed. The identification of this cut-off frequency and the method to eliminating it are also discussed.

II. Analysis

The geometry of a ferrite slab is shown in Fig. 1. The slab is assumed infinite in extent ($x - y$ plane). The bias field is in the y direction and the propagation is in the x direction. The permeability tensor of the biased ferrites is

$$\vec{\mu} = \mu_0 \begin{bmatrix} \mu_{11} & 0 & -j\mu_{12} \\ 0 & 1 & 0 \\ j\mu_{12} & 0 & \mu_{11} \end{bmatrix}, \tag{1}$$

where, including the magnetic loss (the damping term) [12]

$$\mu_{11} = 1 + \frac{(\omega_i + j\omega_r)\omega_m}{(\omega_i + j\omega_r)^2 - \omega^2}, \tag{2}$$

$$\mu_{12} = \frac{\omega\omega_m}{(\omega_i + j\omega_r)^2 - \omega^2}, \tag{3}$$

$$\omega_i = \gamma\mu_0 H_i, \tag{4}$$

$$\omega_r = \gamma\mu_0 \frac{\omega}{2\omega_i}\Delta H, \tag{5}$$

and

$$\omega_m = \gamma\mu_0 M_0, \tag{6}$$

$4\pi M_0$ (Gauss) is the material saturation magnetization, H_i (Oe) is the applied magnetic field, ΔH is the resonance linewidth [12],

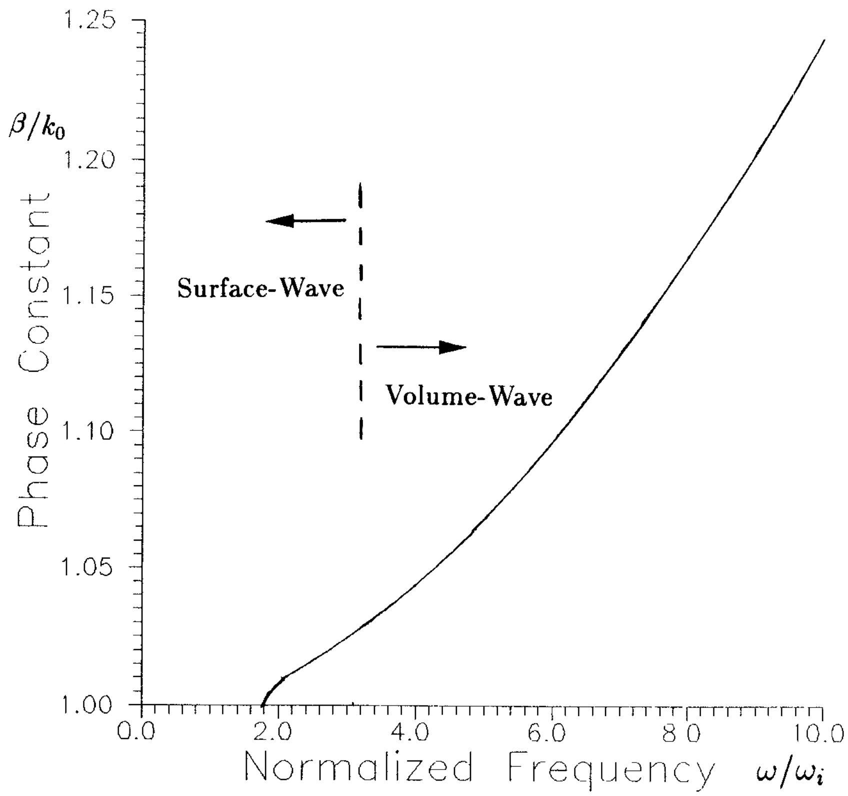

Fig. 2. Dispersion relation for the dynamic (second) mode. H_ι = 500 Oe, 4 πM_0 = 1000 Gauss, ΔH = 10 Oe, d = 0.4 mm, ϵ_f = 14.

and $\gamma\mu_0$ = 1.759 × 10^7 rad/sec/Oe. The eigenvalue equation for the phase constants of TE modes is found from the solution of electromagnetic boundary value problem [6]:

$$2\mu_{11}q_0q_f \coth q_f d = \frac{k^2\mu_{12}^2 - q_f^2\mu_{11}^2 - q_0^2(\mu_{11}^2 - \mu_{12}^2)^2}{\mu_{11}^2 - \mu_{12}^2}, \tag{7}$$

where

$$q_0 = \sqrt{k^2 - k_0^2}, \tag{8}$$

$$q_f = \sqrt{k^2 - k_0^2\epsilon_f\mu_e}, \tag{9}$$

$$\mu_e = \frac{\mu_{11}^2 - \mu_{12}^2}{\mu_{11}} \tag{10}$$

k_0 is the free-space wave number, $k = \beta + j\alpha$ is the complex phase constant of the guided wave, ϵ_f is the dielectric constant of the ferrite slab, and d is the slab thickness.

Numerical solutions of the characteristic equation in (7) provide $\omega - \beta$ diagrams for the propagating modes. With magnetic loss included, Newton methods are used to solve the two-variable nonlinear equations. The fundamental-mode $\omega - \beta$ diagram of a low-loss ferrite slab is shown in Fig. 1. Note that if $Re(q_f^2) < 0$ for a set of ω and k, the mode is a volume wave, and if $Re(q_f^2) > 0$, the mode is a surface wave. It is known that the magnetostatic surface wave exists in the frequency range $\sqrt{\omega_\iota(\omega_\iota + \omega_m)} \le \omega \le \omega_m/2 + \omega_\iota$. The lower and upper frequency limits are referred to as ω_l and ω_u, respectively. The frequency ω_l corresponding to $Re(q_f) = 0$ separates the volume wave and the surface wave. The lossless phase constant at this frequency (β_l) is found to have a simple analytic form ((7) with $\mu_{11} = 0$).

$$\beta_l = k_0\sqrt{1 + \frac{\omega_\iota}{\omega_m}}. \tag{11}$$

For frequency above ω_l, this mode is similar to the magnetostatic surface-wave mode found in a static analysis. The magnetostatic mode properties have been well discussed and are not repeated here. For frequency below ω_l, the mode is a volume wave without cut-off. For very low frequencies, the phase constant increases monotonically but slowly with frequency, even at the ferrite resonance ($\omega = \omega_\iota$), and the mode is similar to the TE_0 mode of a dielectric slab. For frequencies close to but less than ω_l, if the magnetic loss is neglected, it is seen from (7) that when μ_{11} is close to zero the mode discontinuities occur. These mode discontinuities are not physically possible. It is found that for small, but nonzero, magnetic loss, the phase constant is a smooth function of ω near the transition frequency ω_l.

The additional mode found from the full-wave method is shown in Fig. 2 and is referred to as a dynamic mode. The cut-off frequency ω_t of this dynamic mode can be found analytically from (7) with $k = k_0$ (or $q_0 = 0$) as:

$$\omega_t = \sqrt{\omega_\iota\left(\omega_\iota + \frac{\epsilon_f}{\epsilon_f - 1}\omega_m\right)}. \tag{12}$$

It is noted that since ϵ_f is greater than 1, the condition $w_l < \omega_t$ always holds. The magnetostatic surface wave and the dynamic surface wave may exist simultaneously. For the magnetostatic surface-wave applications, this dynamic mode is undesirable and contributes to the losses. In order to push the dynamic mode out of the frequency band of the magnetostatic surface wave, the condition $\omega_u \le \omega_t$ must hold, which corresponds to

$$\left(\frac{\epsilon_f - 1}{4}\right)\omega_m \leq \omega_i. \tag{13}$$

In general, since ϵ_f is quite large, a large bias field is needed to satisfy the above condition. The dynamic mode turns on as a surface wave. As frequency increases, $Re(q_f^2)$ decreases. When frequency becomes greater than ω_s where $Re(q_f^2) = 0$, this dynamic mode becomes a volume-wave (oscillatory) mode. This dynamic mode resembles to the TE_0 mode of a dielectric slab for frequency greater than ω_s. The transition frequency ω_s is found from the following equation ($\Delta H \approx 0$ and $q_f = 0$ in (7)):

$$\mu_e\sqrt{\epsilon_f \mu_e - 1} + \frac{1}{k_0 d} - \sqrt{\left(\frac{1}{k_0 d}\right)^2 + \epsilon_f \mu_e \frac{\mu_{12}^2}{\mu_{11}^2}} = 0. \tag{14}$$

The frequency range where surface waves exist in a ferrite slab is $\omega_l \leq \omega \leq \omega_s$.

III. Conclusion

In this paper, we reexamined the properties of guided-wave modes of a ferrite slab. The phase constant of the fundamental mode in transition from volume to surface wave were identified analytically. It was also found that there is no mode discontinuity as long as the magnetic loss is nonzero. The cut-off frequency of the dynamic mode coexisting with the magnetostatic surface-wave mode was identified analytically. The method of pushing this dynamic mode out of the frequency range of the magnetostatic surface-wave range was described. The frequency range where surface-wave modes of a ferrite slab exist was identified analytically. Outside this frequency band, the ferrite slab is capable of supporting only oscillatory modes.

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